Point-particle system

Objective: Describe what is meant by the "point-particle system"; apply the momentum principle to the point-particle system; apply the energy principle to the point-particle system and real system.

Review

The momentum principle for a multi-particle system is

$$\vec{F}_{net} = \frac{d\vec{P}_{tot}}{dt} \tag{1}$$

where the total momentum of the system is equal to the sum of the momenta of the particles in the system which is also equal to the momentum of the center of mass of the system.

$$\vec{P}_{tot} = \vec{p}_1 + \vec{p}_2 + ... = \vec{p}_{cm} \tag{2}$$

The kinetic energy of a multiparticle system is

$$K_{tot} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + ... \tag{3}$$

which can be broken up into various terms, the translational kinetic energy (the kinetic energy of the system as if it were a point with all of the mass located at the center of mass) and the kinetic energy calculated relative to the center of mass (as if the center of mass were at rest). Therefore,

$$K_{tot} = \frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 + \frac{1}{2}m_3 v_3^2 + ... = K_{trans} + K_{rel} \tag{4}$$

Examples of kinetic energy relative to the center of mass are rotational kinetic energy and vibrational kinetic energy of a diatomic molecule.

Point-particle system

It is convenient to treat a system as a point particle, which we call the point-particle system. Our point particle has these properties:

- The same forces that act on the system act on the point particle.
- A point particle can only have translational kinetic energy; it has no internal energy (such as chemical energy, thermal energy, etc.), no rotational kinetic energy, no vibrational energy (kinetic or potential), etc.
- The point particle is located at the center of mass of the system and has the same mass.
- The translational kinetic energy and the momentum of the point particle is the same as that of the system.

The energy principle applied to the point particle is

$$\Delta K_{trans} = \int_i^f \vec{F}_{net,ext} \cdot d\vec{r}_{cm} \tag{5}$$

Note that the work done on the point particle is the work done by the net force on the point particle. This is NOT necessarily equal to the sum of the work done by each force acting on the real system! Consider the following examples to help you understand this.

Example: stretching a spring

Suppose that you stretch a string by holding each end and pulling the ends apart. Apply the energy principle to the real system consisting of the spring.

Apply the energy principle to the point-particle system (treating the spring as a point particle).

Example: a person jumping from the floor

Suppose that you jump from the floor. Define yourself as the system and apply the energy principle to the real system during the time interval that your feet are still in contact with the floor. You may assume that no energy is transferred between you and your surroundings via thermal energy transfer.

Apply the energy principle to the point-particle system (treating the yourself as a point particle).

Application

1. Two pucks lie on ice and can slide with little friction. A string is attached to each puck and the string is pulled with a constant force F. The string is wound around the outer edge of puck 1 but attached to the center of puck 2. They both start from rest. Try to imagine what you would see as they move. What do you think will happen in the next 3 seconds?

   (a) 2 will move farther than 1

   (b) 1 will move farther than 2

   (c) 1 and 2 will move the same distance

2. A skater pushes straight away from a wall. She pushes on the wall with a force whose magnitude is F, so the wall pushes on her with a force F (in the direction of her motion). As she moves away from the wall, her center of mass moves a distance d. What is the correct form of the energy principle for the real system consisting of the skater?

   (a) $\Delta K_{trans} + \Delta E_{internal} = Fd$

   (b) $\Delta K_{trans} + \Delta E_{internal} = -Fd$

   (c) $\Delta K_{trans} + \Delta E_{internal} = 0$

   (d) $\Delta K_{trans} = Fd$

   (e) $\Delta K_{trans} = -Fd$

3. A skater pushes straight away from a wall. She pushes on the wall with a force whose magnitude is F, so the wall pushes on her with a force F (in the direction of her motion). As she moves away from the wall, her center of mass moves a distance d. What is the correct form of the energy principle for the point particle system consisting of the skater?

   (a) $\Delta K_{trans} + \Delta E_{internal} = Fd$

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   (c) $\Delta K_{trans} + \Delta E_{internal} = 0$

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