1 Introduction

This lab will introduce the concepts of distance modulus and using supernovae as a measuring tool to determine the distances to galaxies beyond our own. Read through this information before proceeding on with the lab.

1.1 Distance Modulus

As we have discussed before, parallax is the apparent shift of an object’s position relative to more distant background objects caused by a change in the observer’s position. In other words, parallax is a perspective effect of geometry. It is the observed location of one object with respect to another—nothing more.

Parallax is measured using the **parsec**. The name parsec comes from “a distance corresponding to a parallax of one arc second.” It was coined in 1913 at the suggestion of British astronomer Herbert Hall Turner. A parsec is the distance from the Sun to an astronomical object which has a parallax angle of one arcsecond (13,600 of a degree). In other words, if a straight line were drawn from the object to the Earth, and another line drawn from the object to the Sun, if the angle formed between the two lines is exactly one arcsecond, then the object’s distance would be exactly one parsec.

(a) A parsec is the distance from the Sun to an astronomical object which has a parallax angle of one arcsecond. (1 AU and 1 pc are not to scale (1 pc ≈ 206,264.81 AU)).

(b) Diagram of the inverse square law and light.

Parallaxes, however, can only be measured for stars out to distances of 500 light-years. Since our galaxy is approximately 100,000 light-years in diameter, this only includes a small fraction of the total number of stars in the galaxy. How can the distances to stars even farther away be determined? One method that can be used is to compare their apparent brightness and luminosity.
Suppose a friend in the distance is carrying a powered 100W light bulb. The further away the friend is, the dimmer the light bulb will appear. The closer the friend is, the brighter the light bulb will appear. So, by comparing how bright the bulb appears to how bright the light bulb is intrinsically, the distance can be determined. This is possible because of the inverse square law for light.

Figure 1(b) visually depicts the inverse square law and light. The number of photons/rays going through each square is different depending on the distance of the square. Like parallax, this is a purely geometric effect.

Astronomers express the inverse square law effect with the distance modulus which is expressed in terms of magnitudes. The difference between the apparent magnitude ($m$) and the absolute magnitude ($M$) defines the distance to the object in parsecs. That is:

$$m - M = -5 + 5 \log_{10} d$$  (1)

Table 1 shows some example values calculated using Equation (1). Note that the 10, 16, 25, 40, 63 pattern repeats (with an increasing number of zeroes) and may be used to calculate values not contained in the table.

<table>
<thead>
<tr>
<th>$m - M$</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>15</th>
<th>20</th>
<th>25</th>
</tr>
</thead>
<tbody>
<tr>
<td>distance (pc)</td>
<td>10</td>
<td>16</td>
<td>25</td>
<td>40</td>
<td>63</td>
<td>100</td>
<td>160</td>
<td>250</td>
<td>400</td>
<td>630</td>
<td>$10^3$</td>
<td>$10^4$</td>
<td>$10^5$</td>
<td>$10^6$</td>
</tr>
</tbody>
</table>

Table 1: Example distance moduli calculated using Equation (1).

One of the best known distance indicators are RR Lyrae Stars. These are pulsating variable stars—stars that change in brightness over time because they are periodically growing larger and smaller much like breathing. These stars pulsate because the release of energy from the outer layers of the star varies over time (due to a layer of partially ionized helium). When this ionized layer is close to the center of the star and hot—it becomes very opaque to the flow of radiation and the radiation pressure pushes it outward. When the ionized layer gets far from the star, it cools off and its opacity decreases. Radiation can now stream through and the layer falls back toward the center of the star.

RR Lyrae stars are very good “standard candles.” These are objects where we have a pretty good idea how intrinsically bright they are. It turns out that all RR Lyrae stars have absolute magnitudes very near $M_V = 0.5$. However, since they are small, faint stars, they cannot be seen at large distances.

Figure 2 shows the variation in the apparent magnitude of the RR Lyrae star VX Her. Note that the average apparent magnitude is about 10.5 (where each pulse reaches its maximum). Thus, the distance modulus for this stars is

$$(m - M) = 10.5 - 0.5 = 10$$

which corresponds to a distance of 1000 pc.

There are many other objects that astronomers use with the distance modulus to obtain distance. They all involve some method by which the astronomer uncovers the value of absolute magnitude $M$ for an object and there are many different approaches used. The apparent magnitude $m$ is then observed to obtain the distance.

1.2 Supernovae

A supernova is a violently exploding star. Supernovae can produce as much energy as an entire galaxy for a short period of time and historically have been visible in the daytime. However, they are rather rare in that astronomers
estimate that only one or two occur each century in the Milky Way galaxy.

Astronomers recognize two main types of Supernovae. Type I supernovae involve a white dwarf that is part of a binary system. A white dwarf is an earth-size ball of carbon and oxygen nuclei that is the end state of low mass stars. The gravity of a white dwarf is opposed by electron pressure and there is an upper mass limit of $1.44M_{\text{Sun}}$ known as Chandrasekhar’s Limit. If material flows rapidly from the binary companion star onto the white dwarf, its mass may exceed $1.44M_{\text{Sun}}$ which causes a supernova. The white dwarf is destroyed in a sudden burst of fusion and no remnant is left behind.

Type II supernovae involve very massive stars at the ends of their lives. These stars fuse progressively more massive nuclei in their cores—C, O, Mg, Ne, Si—and finally a core of iron is formed. Since $^{56}\text{Fe}$ is the most stable nuclei, it is not possible to get any more energy from nuclear fusion and the death of the star is imminent. The core of Fe is crushed to incredibly high temperatures and pressures by the weight of the overlying materials—until it basically disintegrates—and the all of the overlying layers “bounce” off of the core region. This type of supernova is less energetic than the Type I and typically a massive object such as a neutron star or black hole is formed.

Figure 3: Supernova SN2005cs was discovered by Wolfgang Kloehr in 2005. It was a Type II Supernova, here photographed at a magnitude of approximately 14mag.

Because supernovae are such energetic events, astronomers can observe them at great distances. In Figure 3 the supernova is clearly bright enough to be distinguished from its host galaxy.

However, they are brief events often lasting only days and astronomers must work diligently to detect them before they reach their peak brightness and begin to fade. They are only useful as distance indicators if it is possible to calibrate them—to relate their observed brightness profile to absolute magnitudes. Type I supernovae are very uniform—the light curves and spectra for Type I supernovae are all fairly identical—and easy to calibrate since astronomers can calculate the amount of energy produced when $1.44M_{\text{Sun}}$ of C and O nuclei fuse. They are much more useful to astronomers as distance indicators than Type II Supernovae.

2 Supernova Light Curve Fitting Explorer

Open the NAAP Supernova Light Curve Fitting Explorer from this link:

http://astro.unl.edu/naap/distance/animations/snCurveExplorer.html

The red line illustrates the expected profile for a Type I supernovae in terms of Absolute Magnitude. Data from various supernovae can be graphed in terms of apparent magnitude. If the data represents a Type I Supernova, it should be possible to fit the data to the Type I profile with the appropriate shifts in time and magnitude. Once the data fit the profile, then the difference between the absolute magnitude (left-side $y$-axis), $M$, and the apparent magnitude (right-side $y$-axis), $m$, again gives the distance modulus per Equation 1.
As an example load the data for 1995D from the pulldown in the upper right. Grab and drag the data until it best matches the Type I profile. Then click the show horizontal bar checkbox in the upper left. Drag the new horizontal bar on the plot to the peak of the light curve. Read the apparent magnitude from the right $y$-axis and the absolute magnitude from the left $y$-axis, and enter those values into the Distance Modulus Calculator at the bottom as shown in Figure 4. Do not ignore the negative sign on the absolute magnitude. The distance is given as the value $d$ on the far right. In the case of 1995D, its distance modulus comes out to

$$(m - M) = 13.3 - (-19.6) = 32.9$$

which corresponds to a distance of 38 Mpc ($38.0 \times 10^6$ pc).

![Figure 4: Using the NAAP Supernova Light Curve Fitting Explorer to determine the type and brightness of supernova 1995D.](image)

3 Analysis

1. For each of the supernova in listed in Table 2, determine the absolute magnitude ($M$), apparent magnitude ($m$), the distance modulus ($m - M$), and the distance ($d$). If the data points do not fit the Type I profile, then put an $\times$ in that supernova’s row. If the data points almost fit the Type I profile but not quite, consult the InterWeb to determine the type. Supernovae 1994Y and 1995D has been done for you.

Solution: This lab has completely glossed over the sub-types of supernova. For example, Type I can actually be broken up into Type Ia, Ib, and Ic supernovae based on the elements observed in the supernova spectrum. Type II can be broken up into multiple subtypes as well, and these are based on the shape of the supernova spectrum. Figure 5 summarizes this.
Supernovae that involve a white dwarf surpassing the Chandrasekhar Limit (1.44\( M_{\text{Sun}} \)) are actually called Type Ia supernova. Meanwhile, Type Ib and Ic supernovae are actually high mass stars collapsing. These stars have shed (or been stripped of) their outer envelope of hydrogen, and, when compared to the spectrum of Type Ia supernovae, they lack the absorption line of silicon. Compared to Type Ib, Type Ic supernovae are hypothesized to have lost more of their initial envelope, including most of their helium. The two types are usually referred to as stripped core-collapse supernovae.

That said, supernovae 1994I and 1998aq came close to but not quite fit the Type I profile. According to the InterWeb, SN1994I is actually a Type Ic supernova:


SN1998aq was a little tougher, and Wikipedia is less forthcoming, but it does appear to be a Type Ia supernova


Figure 5: Supernova types are characterized by their chemistry and light curves.

2. Why can we not determine the distance to some of the supernova like 1994Y?

Solution: Because they are type II supernova; they are high-mass stars at the end of their lives. Since the final mass of a star that becomes a Type II supernova is not necessarily the same across all Type II supernovae (ranges from 8 \(-\) 150\( M_{\text{Sun}} \)), then the supernova themselves will be different. With a Type I supernova, it is—for us, at least—always going to be a white dwarf with exactly 1.44\( M_{\text{Sun}} \) that explodes. The resulting supernova will always be the same.

3. Based on what you know about the two main different types of supernovae, determine which supernova in Table 2 are Type I and which are Type II.

4. Which Type I supernova (or supernovae) was the closest? How close?

Solution: At first I thought SN1993J was a Type Ia supernova. If it was, then it would be the closest at 10.5 Mpc. But it turns out it’s a Type IIb:


That means SN1998bu is actually the closest at 21.9 Mpc.

5. The conversion of parsecs to light years to meters is

\[
1 \text{ pc} = 3.26 \text{ ly} = 3.09 \times 10^{16} \text{ m}
\]

Use this equivalency statement to calculate how far away the supernova(e) you listed in question 4 is in light years and meters. Show your work. (Hint: don’t forget that 1 Mpc = 10\(^6 \) pc.)
<table>
<thead>
<tr>
<th>Supernova</th>
<th>Apparent Magnitude $m$</th>
<th>Absolute Magnitude $M$</th>
<th>Distance Modulus $(m - M)$</th>
<th>Distance $d$</th>
<th>Supernova Type? I or II</th>
</tr>
</thead>
<tbody>
<tr>
<td>1993J</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>II</td>
</tr>
<tr>
<td>1994Y</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>II</td>
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<tr>
<td>1994I</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>Ic</td>
</tr>
<tr>
<td>1987A</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>II</td>
</tr>
<tr>
<td>1999by</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>×</td>
<td>II</td>
</tr>
<tr>
<td>1995D</td>
<td>13.3</td>
<td>−19.6</td>
<td>32.9</td>
<td>38.0 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1999aa</td>
<td>14.8</td>
<td>−19.6</td>
<td>34.4</td>
<td>75.9 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1994ae</td>
<td>13.0</td>
<td>−19.6</td>
<td>32.6</td>
<td>33.1 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1998bu</td>
<td>12.1</td>
<td>−19.6</td>
<td>31.7</td>
<td>21.9 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1999dq</td>
<td>14.7</td>
<td>−19.6</td>
<td>34.3</td>
<td>72.4 Mpc</td>
<td>Ia</td>
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<tr>
<td>1998aq</td>
<td>12.4</td>
<td>−19.6</td>
<td>32</td>
<td>25.1 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1990N</td>
<td>12.7</td>
<td>−19.6</td>
<td>32.3</td>
<td>28.8 Mpc</td>
<td>Ia</td>
</tr>
<tr>
<td>1999ee</td>
<td>14.8</td>
<td>−19.6</td>
<td>34.4</td>
<td>75.9 Mpc</td>
<td>Ia</td>
</tr>
</tbody>
</table>

Table 2: Fill in the empty fields above using the values you determine from the NAAP Supernova Light Curve Fitting Explorer.
Solution:
\[ 21.9 \times 10^6 \text{ pc} \left( \frac{3.26 \text{ ly}}{1 \text{ pc}} \right) = 7.14 \times 10^7 \text{ ly} \]
\[ 21.9 \times 10^6 \text{ pc} \left( \frac{3.09 \times 10^{16} \text{ m}}{1 \text{ pc}} \right) = 6.77 \times 10^{23} \text{ m} \]

6. Which Type I supernova (or supernovae) was the farthest? How far away?

Solution: I got SN1999aa and SN1999ee both pegged at 75.9 Mpc.

7. Use Equation (2) to calculate how far away the supernova(e) you listed in question 6 is in light years and meters. Show your work.

Solution:
\[ 75.9 \times 10^6 \text{ pc} \left( \frac{3.26 \text{ ly}}{1 \text{ pc}} \right) = 2.47 \times 10^8 \text{ ly} \]
\[ 75.9 \times 10^6 \text{ pc} \left( \frac{3.09 \times 10^{16} \text{ m}}{1 \text{ pc}} \right) = 2.35 \times 10^{24} \text{ m} \]

8. The stellar disk of the Milky Way Galaxy is approximately 100,000 ly (30 kpc) in diameter. Could any of the supernovae in Table 2 have occurred in our own galaxy? Why?

Solution: SN1998bu was the closest supernova in Table 2 at 7.14 \times 10^7 \text{ ly away. It would take about 1000 Milky Way Galaxies laid end-to-end to reach SN1993J. For SN1999aa, at 2.47 \times 10^8 \text{ ly away, we would need 10,000 Milky Way Galaxies laid end-to-end to reach it.}\
It is quite safe to say that all the Type Ia supernovae in Table 2 are well outside our galaxy.