A Few Mathematical Skills

1 Determining the number of significant figures

Two basic cases on whether zeros count or not:

1. Zeros that follow a non-zero digit but aren’t followed by a decimal are not counted.

   - $100100 \rightarrow 4$ sigfigs
   - $100100. \rightarrow 6$ sigfigs

2. All zeros to the left of the decimal means that no zeros count unless they are to the right of both the decimal point and a non-zero digit.

   - $0.005 \rightarrow 1$ sigfigs, just the 5
   - $0.00500 \rightarrow 3$ sigfigs, starting at the 5 and going right

Examples

- $1 \rightarrow 1$ sigfig
- $10 \rightarrow 1$ sigfig, no decimal, so zero is meaningless
- $10. \rightarrow 2$ sigfigs, everything between the first non-zero digit and the decimal
- $1001 \rightarrow 4$ sigfigs, everything between the non-zero digits
- $1001. \rightarrow 4$ sigfigs, everything between the first non-zero digit and the decimal
- $100.0 \rightarrow 4$ sigfigs, with non-zero digit to the left of the decimal, everything after the decimal counts
- $100.1 \rightarrow 4$ sigfigs, with non-zero digit to the left of the decimal, everything after the decimal counts
- $0.0001 \rightarrow 1$ sigfig, zero digit to the left of the decimal, only digits after the first non-zero digit on the right count
- $0.000100 \rightarrow 3$ sigfigs, zero digit to the left of the decimal, only digits after the first non-zero digit on the right count

Addition and Subtraction

When adding/subtracting, the answer should have the same number of decimal places as the limiting term. The limiting term is the number with the least decimal places.

1. $6.22 + 53.6 + 14.311 + 45.00901 = 119.22191$

   The number with the least number of decimal places, 53.6, has one decimal place, so round to 119.2.

2. $5365.999 - 234.66706 = 5131.33194$

   The number with the least number of decimal places, 5365.999, has three decimal places, so round to 5131.332
Multiplication and Division

When multiplying/dividing, the answer should have the same number of significant figures as the limiting term. The limiting term is the number with the least number of significant figures.

1. \(503.29 \times 6.177 = 3108.82233\)
   The number with the least significant figures, 6.177, has four sig-figs, so round to 3109.

2. \(1000.1 \div 243 = 4.11563786\)
   The number with the least significant figures, 243, has three sig-figs, so round to 4.12

Conversions

When converting a number, the answer should have the same number of significant figures as the number started with.

\[52.4 \text{ in} \times \left(\frac{1 \text{ ft}}{12 \text{ in}}\right) = 4.366666667 \rightarrow \text{rounds to 4.37 ft}\]

Practice: How many significant figures do the following numbers have?

1. 1234
2. 0.023
3. 890
4. 91010
5. 9010
6. 1090.010
7. 0.00120
8. 0.00390
9. 1020010
10. 780.
11. 1000
12. 918.010
13. 0.0001
14. 0.00390
15. 8120
16. 72

2 Powers of 10

Powers of 10 indicate how many times to multiply 10 by itself.

\[10^2 = 10 \times 10 = 100\]
\[10^6 = 10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000\]

Negative powers are the reciprocals of the corresponding positive powers.

\[10^{-2} = \frac{1}{10^2} = \frac{1}{10} \times \frac{1}{10} = \frac{1}{100} = 0.01\]
\[10^{-6} = \frac{1}{10^6} = \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} \times \frac{1}{10} = \frac{1}{1,000,000} = 0.000001\]

The powers of 10 follow two basic rules:

1. A positive exponent tells how many zeros follow the 1. For example, \(10^0\) is a 1 followed by no zeros, and \(10^8\) is a 1 followed by eight zeros.
2. A negative exponent tells how many places are to the right of the decimal point, including the 1. For example, \(10^{-1} = 0.1\) has one place to the right of the decimal point; \(10^{-6} = 0.000001\) has six places to the right of the decimal point.
Addition and Subtraction

To add or subtract powers of 10, the values must be written in longhand notation.

\[ 10^6 + 10^2 = 1,000,000 + 100 = 1,000,100 \]

\[ 10^8 + 10^{-3} = 100,000,000 + 0.001 = 100,000,000.001 \]

\[ 10^7 - 10^3 = 10,000,000 - 1,000 = 9,999,000 \]

Multiplying and Dividing

Multiplying powers of 10 simply requires adding exponents.

\[ 10^4 \times 10^7 = 10,000 \times 10,000,000 = 100,000,000,000 = 10^{11} \]

\[ 10^5 \times 10^{-3} = 100,000 \times 0.001 = 100 \]

\[ 10^{-8} \times 10^{-5} = 0.00000001 \times 0.00001 = 0.000000000001 = 10^{-13} \]

Dividing powers of 10 requires subtracting exponents.

\[ \frac{10^5}{10^3} = 100,000 \div 1,000 = 100 = 10^2 \]

\[ \frac{10^3}{10^5} = 1,000 \div 10,000,000 = 0.00001 \]

\[ \frac{10^{-4}}{10^{-6}} = 0.0001 \div 0.000001 = 100 = 10^2 \]

Powers of Powers of 10

We can use the multiplication and division rules to raise powers of 10 to other powers or to take roots.

\[ (10^4)^3 = 10^4 \times 10^4 \times 10^4 = 10^{4+4+4} = 10^{12} \]

Note that we can get the same end result by simply multiplying the two powers.

\[ (10^4)^3 = 10^{4 \times 3} = 10^{12} \]

Because taking a root is the same as raising to a fractional power (e.g. the square root is the same as the \( \frac{1}{2} \) power; the cube root is the same as the \( \frac{1}{3} \) power, etc.), we can use the same procedure for roots.

\[ \sqrt{10^2} = (10^4)^{1/2} = 10^{4 \times (1/2)} = 10^2 \]

Practice: Solve the following powers of 10 problems.

1. \( 10^3 + 10^{-1} \)
2. \( 10^{-2} - 10^{-4} \)
3. \( 10^{-4} \times 10^{-1} \)
4. \( 10^5 \div 10^{-5} \)
5. \( (10^7)^{-2} \)
6. \( \sqrt[4]{10^{-2}} \)
3 Scientific Notation

Scientific notation is a way of writing very large or very small numbers. With scientific notation, very large or small numbers are expressed as a product of a number between one and ten and a power of ten. For example, Avogadro's number, often used in chemistry, can be written as:

\[ 602,000,000,000,000,000,000,000 \]

in numeral form, or

\[ 6.02 \times 10^{23} \]

in scientific notation. Newton’s gravitational constant is

\[ 0.00000000006673848 \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]

in numeral form, or

\[ 6.673848 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]

in scientific notation. It’s good form to limit scientific notation to 2 or 3 significant figures, which makes Newton’s gravitational constant become

\[ 6.67 \times 10^{-11} \text{ m}^3 \text{ kg}^{-1} \text{ s}^{-2} \]

To understand how to convert numbers to scientific notation, follow these rules:

1. Scientific notation is a number from 1 to 9 followed by a decimal and the remaining significant figures and an exponent of 10 to hold place value.

\[
5.43 \times 10^2 = 5.43 \times 100 = 543 \\
8.65 \times 10^{-3} = 8.65 \times .001 = 0.00865
\]

2. When the decimal is moved to the left the exponent on the 10 gets larger. When the decimal is moved to the right the exponent on the 10 gets smaller. In both cases the value of the number stays the same. Each place the decimal moves changes the exponent by one.

\[ 6000 \times 10^0 = 600.0 \times 10^1 = 60.00 \times 10^2 = 6.000 \times 10^3 = 6000 \]

(Note that \(10^0 = 1\).)

All the previous numbers are equal, but only \(6.000 \times 10^3\) is in proper scientific notation.

3. To add/subtract in scientific notation, the exponents must first be the same.

\[
(3.0 \times 10^2) + (6.4 \times 10^3)
\]

Since \(6.4 \times 10^3\) is equal to \(64. \times 10^2\), you can swap the latter in and then add.

\[
(3.0 \times 10^2) + (64. \times 10^2) = 67.0 \times 10^2
\]

\(67.0 \times 10^2\) is mathematically correct, but a number in standard scientific notation should have only one number to the left of the decimal, so the decimal is moved to the left one place and one is added to the exponent.

Also, following the rules for significant figures, the answer becomes \(6.7 \times 10^3\).
4. To multiply, find the product of the numbers, then add the exponents.

\[(2.4 \times 10^2) \times (5.5 \times 10^{-4}) = (2.4 \times 5.5) \times (10^2 \times 10^{-4})\]
\[= (13.2) \times 10^{2+(-4)}\]
\[= 13.2 \times 10^{-2} = 1.3 \times 10^{-1}\]

5. To divide, find the quotient of the number and subtract the exponents.

\[\frac{(3.3 \times 10^{-6})}{(9.1 \times 10^{-8})} = (3.3/9.1) \times (10^{-6}/10^{-8})\]
\[= (0.36) \times 10^{-6-(-8)}\]
\[= 0.36 \times 10^2 = 3.6 \times 10^1\]

Practice: Convert these numbers to scientific notation with three significant figures.

1. 287
2. 840,000
3. 0.0000683
4. 603,400,000
5. 0.0000006
6. 5.8
7. 5,100,000
8. 0.009003
9. 0.01957

Practice: Convert these to numeral form while maintaining the number’s original number of significant figures.

1. 1.02 \times 10^{-4}
2. 8 \times 10^5
3. 7.32 \times 10^3
4. 5.59 \times 10^{-3}

4 Working With Units

Units are part of a measurement that tells us what scale or standard is being used to represent the results of the measurement. The International System (SI units) replaced the Metric System as the global system of units in 1960. It’s based on the metric system and units derived from the metric system.

Fundamental SI Units

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Name of Unit</th>
<th>Abbreviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>volume</td>
<td>liter</td>
<td>L</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>temperature</td>
<td>kelvin</td>
<td>K</td>
</tr>
</tbody>
</table>

Commonly Used Prefixes

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Meaning</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>giga</td>
<td>G</td>
<td>1,000,000,000</td>
<td>10^9</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>1,000,000</td>
<td>10^6</td>
</tr>
<tr>
<td>kilo</td>
<td>k</td>
<td>1,000</td>
<td>10^3</td>
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<td>centi</td>
<td>c</td>
<td>0.01</td>
<td>10^{-2}</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>0.001</td>
<td>10^{-3}</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>0.000001</td>
<td>10^{-6}</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>0.000000001</td>
<td>10^{-9}</td>
</tr>
</tbody>
</table>
Measurements of Length, Volume, and Mass

1. **Length**: The fundamental SI unit of length is the meter. \((1 \text{ m} = 3.2808 \text{ ft})\)

   - kilometer \(\text{km}\) \(1000 \text{ m} = 10^3 \text{ m}\)
   - meter \(\text{m}\) \(1.0 \text{ m}\)
   - decimeter \(\text{dm}\) \(0.1 \text{ m} = 10^{-1} \text{ m}\)
   - centimeter \(\text{cm}\) \(0.01 \text{ m} = 10^{-2} \text{ m}\)
   - millimeter \(\text{mm}\) \(0.001 \text{ m} = 10^{-3} \text{ m}\)
   - micrometer \(\mu\text{m}\) \(0.000001 \text{ m} = 10^{-6} \text{ m}\)
   - nanometer \(\text{nm}\) \(0.000000001 \text{ m} = 10^{-9} \text{ m}\)

2. **Volume**: The amount of three-dimensional space occupied by a substance. Each dimension of the volume is measured using the meter.

   - 1 m \(\times\) 1 m \(\times\) 1 m = \((1 \text{ m})^3 = 1 \text{ m}^3\)
   - liter \(\text{L}\)
   - milliliter \(\text{mL}\) \(\frac{1}{1000} \text{ L} = 10^{-3} \text{ L} = 1 \text{ mL} = 1 \text{ cm}^3\)

3. **Mass**: The quantity of matter present in an object. The metric system used the gram as the fundamental unit for mass. It was replaced by the kilogram in the SI unit system.

   - kilogram \(\text{kg}\) \(1000 \text{ g} = 10^3 \text{ g} = 1 \text{ kg}\)
   - gram \(\text{g}\) \(1 \text{ g}\)
   - milligram \(\text{mg}\) \(0.001 \text{ g} = 10^{-3} \text{ g} = 1 \text{ mg}\)

### 5 Unit Conversion

Converting between different units can be made simple by using Figure 1 to turn the conversion into a puzzle game.

1. Identify the unit we have and the unit we want to convert it to.
   
   For example, say we want to know how tall a 5 ft 10 in person is in centimeters. The units we’re given are feet and inches and we want to know what this is in centimeters.

2. Find the **conversion factor** that gives a ratio between the two units.
   
   In this example, we start with feet and inches and we want to convert that to centimeters. Since feet and inches are different units themselves, we’ll have to convert feet to inches first before converting everything to centimeters. We’ll need these two **equivalence statements**:

   
   \[
   \begin{align*}
   1 \text{ foot} &= 12 \text{ inches} \\
   1 \text{ inch} &= 2.54 \text{ cm}
   \end{align*}
   \]

   These equivalence statements have infinite significant figures.

3. Plug your given measurement and the conversion factor into Figure 1 and calculate the measurement in the new unit.

   For this case, we need to convert 5 feet to inches using the first equivalence statement above:

   \[
   5 \text{ feet} \times \left( \frac{12 \text{ inches}}{1 \text{ foot}} \right) = \text{ inches}
   \]

   Cancel units and then do the arithmetic:

   \[
   5 \text{ feet} \times \left( \frac{12 \text{ inches}}{1 \text{ foot}} \right) = 60 \text{ inches}
   \]
Add these 60 inches to the remaining 10 inches, and then repeat the process to convert inches to centimeters.

\[
70 \text{ in} \times \left( \frac{2.54 \text{ cm}}{1 \text{ in}} \right) = 177.8 \text{ cm} = 180 \text{ cm}
\]

Figure 1: This is from http://www.youtube.com/watch?v=XKCZnSMLKvk

These conversions can also be chained together into one long expression. The distance from High Point to Baltimore is 372 miles along I-85 and I-95. If I want to know how many smoots that is, I can use the following equivalence statements to get the answer:

1 mile = 1.6 km 
1 km = 1000 m 
1 m = 0.5876 smoots

The conversion looks like this:

\[
372 \text{ mi} \times \left( \frac{1.6 \text{ km}}{1 \text{ mi}} \right) \times \left( \frac{1000 \text{ m}}{1 \text{ km}} \right) \times \left( \frac{0.5876 \text{ smoot}}{1 \text{ m}} \right) = 349739 \text{ smoot}
\]

\[
= 350000 \text{ smoot} = 3.50 \times 10^5 \text{ smoot}
\]

Note that the organizing principle here is just to cancel all the units except the one unit we're converting to, which, in this case, is smoots. As long as we have an equivalence statement, we can convert any unit to any other unit with this process.

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1https://en.wikipedia.org/wiki/Smoot