Momentum

Objective: Describe evidence of an interaction; describe the motion of a particle in the absence of interactions (or the canceling of the effects of various interactions); describe the relativity principle and why it is important; define momentum; define the momentum principle.

Types of interactions
In order of decreasing strength:

1. strong
2. electromagnetic
3. weak
4. gravitational

A common kind of interaction is “contact”. How would this kind of interaction be classified?

Evidence of interactions
The most common indicators of interactions are:

- change of velocity (change of direction and/or change of speed)
- change of identity
- change of shape of multiparticle system
- change of temperature of multiparticle system
- lack of change when change is expected

Newton’s first law of motion – “the natural state of a particle”
In the absence of interactions or if all interactions cancel each other’s effects so that the result is the same as if there were no interactions, what is the motion of a particle? Newton’s first law of motion says:

An object moves in a straight line and at constant speed except to the extent that it interacts with other objects.

The answer to my question is, uniform motion (or constant velocity) – that is the natural state of things. One possible case is that the particle remains at rest. However, any constant velocity is allowed.

Suppose that a space ship is at rest in space very far from any large body. It then fires its thrusters which cause it so speed up in the +x direction until it reaches a speed of 1000 mi/h. At this instant, it shuts off its thrusters. Describe the motion of the space ship.

Suppose a car is traveling at a constant velocity of 60 mi/h on a level, straight road when the driver puts it in neutral. Describe the motion of the car. Does this violate Newton’s first law?

Relativity
Is Newton’s first law valid for all reference frames? For example, if I am riding in a spaceship and do an experiment like tossing a ball, will I get the same results as I would on Earth? That depends on the motion of the spaceship and Earth. The principle of relativity states that

Physical laws work in the same way for observers in uniform motion as for observers at rest.

An observer at rest and an observer motion at constant motion (with respect to the first) will agree on the physical laws. However, if the observer in motion is changing speed or direction, they will NOT agree.

Ok, but who is really at rest? The Earth is moving (rotating as well as orbiting about the Sun), the Sun is moving within the galaxy, our galaxy is moving relative to other galaxies, etc. Is anyone really at rest and how do we know if we are moving? After all, maybe we are at rest and everything else is moving relative to us.
First of all, it doesn’t matter who is at rest and who is moving because there is no way to tell the difference—after all the laws of physics are the same. What we need is a reference frame in which Newton’s first law is valid—this is called an **inertial reference frame**. Then, any reference frame moving at constant velocity relative to that one is also an inertial reference frame.

Is the Earth an inertial reference frame? No, well yes, but only approximately an inertial reference frame. If you measure things over small time intervals, then the motion of the Earth relative to the Sun (and to distant stars) is approximately uniform.

**Momentum**

Our next goal is to relate an interaction to the effect of an interaction on a particle’s motion. The interaction required to toss a tennis ball is clearly different than the interaction required to toss a bowling ball. The mass of the object, as well as the velocity of the object, is important in describing an object’s motion and its response to an interaction. Mass can be defined in a number of different ways. One loose definition is that mass is the amount of “stuff” an object is made of—the more atoms, the more mass. However, different atoms have different amounts of mass, but one again, the more protons and neutrons an atom has, the more mass it has.

A natural way to define “motion” is thus \( \vec{m} \vec{v} \), a quantity called **momentum**. However, Einstein discovered that the mass of an object depends on how fast it is moving—the faster the object is moving, the more massive it is. This is called its relativistic mass.

\[
\text{relativistic mass} = \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}
\]

where \( m \) is called the **rest mass** of the particle. Therefore, the momentum of a particle is its relativistic mass times its velocity:

\[
\text{momentum} \vec{p} = \frac{m\vec{v}}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}}
\]

Written in terms of its components:

\[
< p_x, p_y, p_z > = \left( \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \right) < v_x, v_y, v_z >
\]

Since this is a vector equation, you can equate the components of the vectors.

\[
p_x = \left( \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \right) v_x
\]

\[
p_y = \left( \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \right) v_y
\]

\[
p_z = \left( \frac{m}{\sqrt{1 - \frac{|\vec{v}|^2}{c^2}}} \right) v_z
\]

**Application**

1. If the speed of an object is 0.1c, what is the factor \( \frac{1}{\sqrt{1 - |\vec{v}|^2/c^2}} \)? What about for 0.9c? What about for 0.9999c?

2. What is the magnitude of the momentum of a baseball (155 grams) moving at a speed of 45m/s?

3. What is the momentum of a 80-kg sprinter who runs 100 m in 10 s? Assume he is running at constant velocity in the +x direction.

4. What is the momentum of the Hubble Space Telescope if its mass is 11 Mg, its speed is \( 2.8 \times 10^4 \text{ km/h} \), and its direction is \( 0.200,0.400,0.894 \)¿? What is the magnitude of its momentum? What is its speed?