Collisions

Objective: Describe what is meant by “scattering”; apply the momentum principle and energy principle to a collision where the incoming particle is scattered at some deflection angle;

Scattering

When an incoming particle collides with a stationary particle, they are each deflected. The incoming particle is “scattered” through an angle $\theta$, and the target particle is deflected an angle $\phi$. Since the net external force on the system is zero, then the momentum of the system is conserved. If there are no internal changes in the energies of the particles, then the energy of the system is also conserved. Since the energy of the system well before and well after the collision is kinetic energy, then we can just say that the kinetic energy of the system is conserved. These two fundamental principles lead to

$$\vec{p}_1 = \vec{p}_3 + \vec{p}_4 \quad (1)$$

$$K_1 = K_3 + K_4 \quad (2)$$

If the deflection angle of the incoming particle (relative to the direction of its initial momentum) is $\theta$ and the deflection angle of the target particle (relative to the direction of the initial momentum of the incoming particle) is $\phi$, then conservation of momentum and conservation of energy give

$$p_1 = p_3 \cos(\theta) + p_4 \cos(\phi) \quad (3)$$

$$0 = p_3 \sin(\theta) - p_4 \sin(\phi) \quad (4)$$

$$\frac{p_1^2}{2m} = \frac{p_3^2}{2m} + \frac{p_4^2}{2M} \quad (5)$$

Typically in a lab, you measure $p_1$ and the angle the incoming particle was scattered as a result of the collision.

Impact parameter

The scattering angle depends on the impact parameter which is the perpendicular distance between the target particle and the direction of the momentum of the incoming particle. Imagine that incoming particles with random impact parameters are fired at the target particle. The scattering angle for those that have small impact parameters will be much larger than the scattering angle of those that have large impact parameters. How the impact parameter affects the scattering angle depends on the interaction (i.e. force) between the particles.

Inelastic collisions

In some types of collisions, the objects will stick together. For example, a velcro covered cart on a track will stick to another velcro covered “target” cart when they collide. After the collision, the carts move off as one object. In this collision, there is a change in the internal energy of the system and as a result, the kinetic energy of the system is not conserved. Note that you can still apply the energy principle but you would have to measure the change in the internal energy (such as thermal energy) or changes in other such energies in the system.

In some collisions between subatomic particles, there is a change in the identity of one or both particles. For example, a collision between a pion ($\pi^-$) and a proton produces a scattered pion and a delta ($\Delta^+$. Conservation of momentum and energy apply, though kinetic energy is not conserved. Another example is the excitation of a nucleus during a collision. If you shoot alpha particles at a gold nucleus with high enough initial energy, then the alpha particles can excite the nucleus of the golf atom into a higher energy state. Though energy will be conserved for the system, kinetic energy is not conserved.

In the explosion of a shell, momentum and energy are conserved; however, the kinetic energy is not conserved. During the explosion, there is a change in chemical energy which results in an increase in kinetic energy, an increase in thermal energy, etc.

Scattering experiments are essential for discovering new particles and for understanding the excited states of quark systems (i.e. particles made up of quarks, like the proton).
1. A 21.0 kg body is moving in the direction of the positive x axis with a speed of 220 m/s when, owing to an internal explosion, it breaks into three parts. One part, whose mass is 8.0 kg, moves away from the point of explosion with a speed of 80 m/s in the direction of the positive y axis. A second fragment, with a mass of 6.00 kg, moves in the direction of the negative x axis with a speed of 525 m/s. What is the velocity of the third (7.00 kg) fragment? How much energy was released in the explosion?

2. A proton with a speed of 500 m/s collides elastically with another proton initially at rest. The projectile and target protons then move along perpendicular paths, with the projectile path at 30° from the original direction. What is the speed of the target proton after the collision? What is the speed of the projectile proton after the collision?

3. In a game of pool, the cue ball strikes another ball of the same mass and initially at rest. After the collision, the cue ball moves at 3.70 m/s along a line making an angle of 18.0° with its original direction of motion, and the second ball has a speed of 1.60 m/s. Find the angle between the direction of motion of the second ball and the original direction of motion of the cue ball. Find the original speed of the cue ball. Is kinetic energy conserved? (Of the centers of mass, disregard the rotation.)

4. A billiard ball moving at a speed of 2.2 m/s strikes an identical stationary ball a glancing blow. After the collision, one ball is found to be moving at a speed of 0.9 m/s in a direction making a 45° angle with the original line of motion. Find the velocity of the other ball. Is the collision elastic or inelastic?

5. A railroad freight car weighing 284 kN and traveling at 1.50 m/s overtakes one weighing 206 kN and traveling at 0.912 m/s in the same direction. The cars couple together. Find the speed of the cars after collision. Find the loss of kinetic energy during collision.